Map Projections for the Graphical Representation of Spherical Measurement Data

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Introduction

Many tasks in acoustics and audio engineering entail capturing spatial measurement data on a sphere and require an appropriate graphical representation of this data. To represent a circular dataset the polar plot is a very popular diagram type. The corresponding diagram type for a spherical dataset is a three-dimensional balloon plot. However, the latter requires a three-dimensional illustration to represent the full dataset simultaneously. Actual screens and print media usually only represent two dimensions. The balloon must be rotated or be printed in different angles in order to achieve a full dataset overview. A very similar problem arises in cartography where e.g. the spherical earth surface must be reduced to two-dimensional plane maps. Cartographers developed several map projections and some of the proposals are very convenient to solve the problem of three-dimensional measurement data representation.

Map Projections

A map projection is a systematic representation of a curved surface on a flat plane. Map projections are the subject of the (mathematical) cartography and are generally used to represent the curved earth surface on a map. The topic of map projections at least dates back

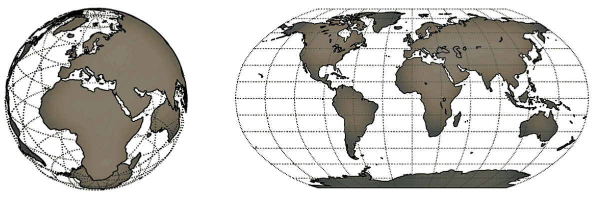

Figure 1: Globe and Robinson projection of a world map.

to the time of the Greek mathematician, geographer, astronomer and philosopher Claudius Ptolemy (Latin: Claudius Ptolemaeus), * c. 90 AD – † c. 168 AD [1]. Most of the currently known and applied projections have been developed during the 16th to 19th centuries and are named after their inventors. Further variations have been proposed during the 20th century [2]. Even if very few map projections are projections in a strict sense, most of them are only defined in terms of mathematical formulae or even reference points and do not have a straight physical interpretation. Map projections need a developable target surface like a cylinder, a cone or a plane and can be categorized by their target surfaces leading to e.g. cylindric, pseudo-cylindric, conic or azimuthal projections.

Distortion

Within his *Theorema Egregium* [3], Carl Friedrich Gauss (*1777 – †1855) proved that the representation of a sphere on a plane always entails distortion. Some map projections are designed to minimize distortions of certain metric properties at the cost of maximizing errors in others. A map projection can be designed to preserve the best possible conformality, distances, directions, scales or areas. But it is not possible to preserve all of these properties at the same time. Map projections are often categorized by their metric properties leading to groups of e.g. conformal, equidistant or equal-area projections. The french mathematician and cartographer Nicolas Auguste Tissot (*1824 – †1897) proposed the Tissot Indicatrix to reveal and rate the distortions caused by a map projection [4]. The Tissot Indicatrix is a circle of infinitesimal radius on the spherical surface transforming into an ellipse indicating area, angular and linear distortions for a single point. Several of these Indicatrices are distributed over the map to show the distortion across the whole surface, as shown in Figure 2.

Suitable Projection Types

Although hundreds of map projections are available, not all of them are equally suitable for the presented application. Spherical measurements and the subsequent evaluations are often used to acquire and represent spatial energy distributions. To give a comprehensive overview of the spatial distribution for all directions, equal-area projections are convenient for example. A very important detail is the representation of the polar regions. Many projections tend to severely exaggerate the size of the polar regions, see Fig. 3 (a) and (b). This is not suitable for a meaningful overview of all directions. Hence the number of appropriate projections for a full-space data representation is quite restricted. Equal area projections with e.g. elliptical output plots lead to useful results. For example Mollweide or Hammer-Aitoff projections turn out to have useful characteristics and lead to convincing results.

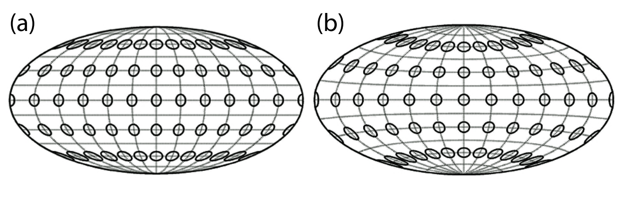

Figure 2: Tissot Indicatrix [4] applied to (a) Mollweide and (b) Hammer-Aitoff projections to indicate distortions.

Examples and Illustrations

This section provides examples to illustrate the use of map projections representing acoustic measurement data. The first example is taken from from sound field analysis using microphone arrays, see Fig. 3. This example simultaneously is used to quickly figure out some important characteristics of the map projections. In contrast to carthography the angles for longitude and latitude are defined as $\phi \in [0,360[^\circ$ and $\theta \in [0,180]^\circ$.

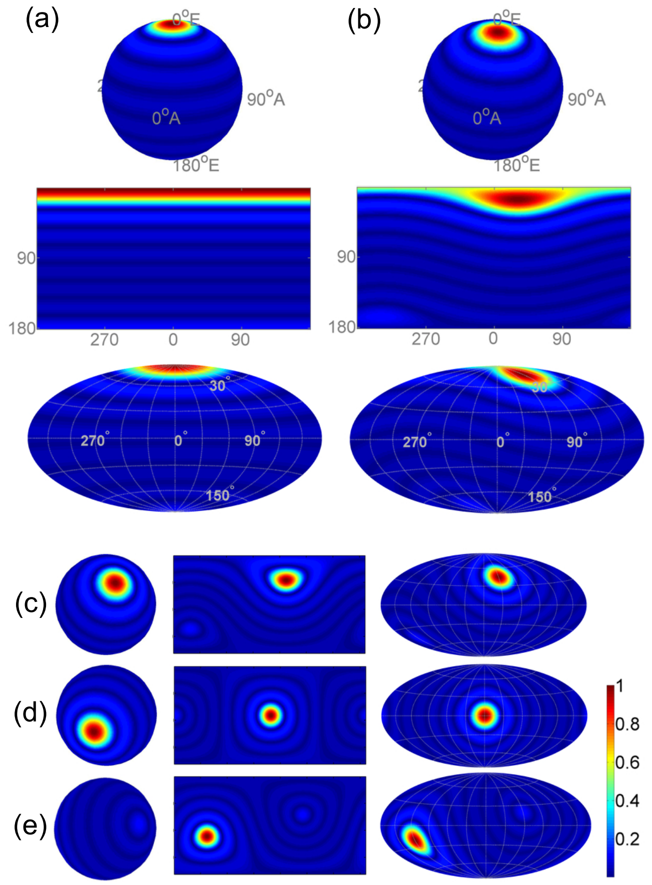

Figure 3: Simulated plane wave impacts from (a) $\phi = 30^\circ$, $\theta = 0^\circ$, (b) $\phi = 30^\circ$, $\theta = 15^\circ$, (c) $\phi = 30^\circ$, $\theta = 45^\circ$, (d) $\phi = 0^\circ$, $\theta = 90^\circ$, (e) $\phi = 240^\circ$, $\theta = 110^\circ$ to a spherical microphone array using a decomposition of order N=7. The images show the originating 3D globe plots, a very simple cylindrical Plate Carrée projection and a Hammer-Aitoff projection with an elliptical target image. The globe plot cannot reveal the full spatial response. (e) The simple cylindrical projection leads to a strong distortion around the polar regions (a),(b). The Hammer-Aitoff projection delivers a very good image for the entire space.

The second example illustrates the representation of loudspeaker directivities at different frequencies using map projections compared to commonly used balloon plots, see Fig. 4. The map projections provide a good overview on the entire spatial distribution. Directions and intensities for the full dataset can directly and clearly be read off the image. The balloon plots do not represent the full spatial dataset, which may be a disadvantage for observing non-symmetric responses. Furthermore it is more difficult to judge the entire spatial energy distribution and to read off directions and intensities.

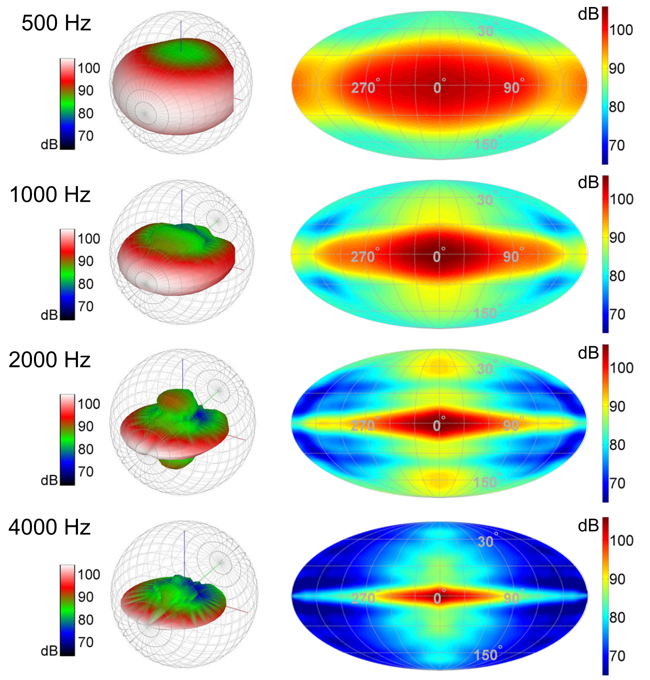

Figure 4: Ease balloons (left) and Mollweide projections (right) of a short line source speaker at different frequencies.

Conclusion

A proposal for the representation of spherical measurement data based on map projections originating from cartography has been presented. Convenient projection types have been elaborated and shown. The methods are useful in order to deliver a complete overview of full three-dimensional datasets and can generally be applied to any kind of spherical measurement data. In acoustics typical applications can be found in sound field analysis or the visualization of radiation patterns e.g. for instruments or loudspeakers. The resulting images are a convincing alternative e.g. to the commonly used balloon or globe plots.

Funding

The work is funded by the Federal Ministry of Education and Research in Germany. Code 17009X11-MARA.

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