Efficiently adding sampling points for improving HRTF measurements on arbitrary grids

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Introduction

A head-related transfer function (HRTF) is a filter to describe the sound incidence from a source, which is typically in the far-field, to the left and right ear, incorporating both monaural and the binaural cues. For a full-spherical representation of sound incidence, it is necessary to acquire a large number of HRTFs for any location on a sphere surrounding the head. Even though it is possible to use a generic HRTF set, the use of an individual HRTF set can increase the localization where the interaural time difference is low, e.g. on the saggital plane, and individual spectral cues are important for localization[Wen93].

There are several ways to measure such individual full-spherical HRTF datasets, most of them require a large amount of equipment and effort[Bri19][Ric19]. To make such measurements available to a broader audience, e.g., to institutions without access to anechoic chambers or measurement robotics, one possible approach could be a simple self-guided measurement setup. In this approach, the HRTF is captured conventionally as an impulse response with a loudspeaker and in-ear microphones. While the loudspeaker is fixed in position, the subject is free to rotate his head and trigger measurements by choice with a remote control. To associate a measurement with the correct angle of sound incidence, the subjects head orientation is traced by a head-tracker. Afterwards, the measurements can be further post-processed, e.g., to compensate for head movement errors or room reflections.

While this approach is quite easy to perform, it is hard to get a even distributed full-spherical measurement. I.e., it is not trivial to measure all elevation positions on the lateral plane without extensively tilting the head. At some point during the measurement, the user needs some instructions that guide him explicitly to necessary angles that might have been left out. A good compromise between user freedom and guidance would be a two-step approach where in the first step, the user is free to perform some measurements. In the second step, an algorithm computes and proposes some additional measurement positions based on the existing measurements.

In this work, we focus on such an algorithm to find additional points to an existing random point set on the sphere. First, the basic principle of the algorithm is outlined. Then, we focus on the core elements of the algorithm, these are the pre-selection stage and the testing stage, as explained in the overview. Afterwards the results are evaluated and discussed.

Algorithm Overview

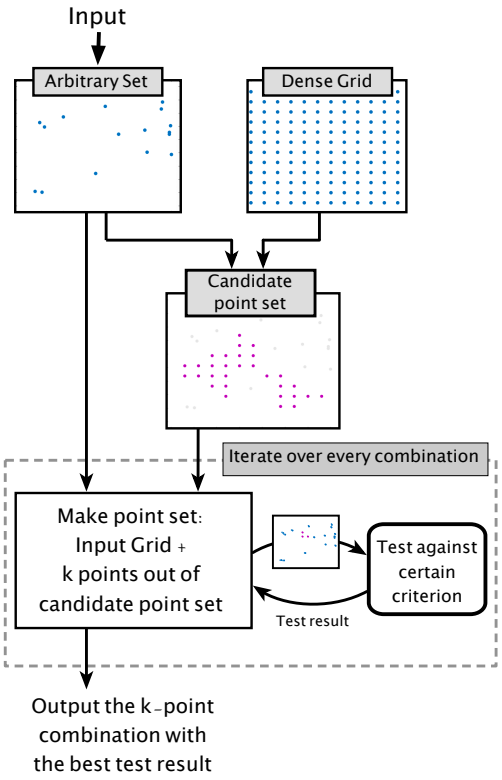


Figure 1: Schematic outline of the algorithm

Considering a given arbitrary set of L sampling positions as spherical angles $\vec{\Omega}_L = \{\Omega_1, \cdots, \Omega_L\}$ with $\Omega \equiv (\phi, \theta) \in S^2$, where ϕ is the azimuth and θ the elevation. To determine the optimal placement for K additional sampling positions, we need to test step-by-step every possible combination of K positions out of a candidate position set $\vec{\Omega}_Q$.

First, we need to define a suitable candidate point set out of a dense basis grid by identifying the areas where the input point set has significant gaps. This step is described in detail in the following section *Pre-selection of point candidates*.

Then, in a repetitive process, a combination of K points is picked from the candidate point set and is added to the input point set. This joint point set $\{\vec{\Omega}_L \vec{\Omega}_Q\}$ is tested against a certain criterion to determine the improvement regarding an even point distribution. This is repeated for every possible combination out of the candidate point set. To measure the improvement of a grid, a suitable test has to be applied. Two variants of such a test criteria

are presented in the following sections *Test variant 1: Matrix condition number* and *Test variant 2: Minimum Point Distance*.

To test every combination of K positions out of the candidate point set $\vec{\Omega}_Q$ with Q positions, we have to iterate over every possible subset $\vec{\Omega}_K \subset \vec{\Omega}_Q$ that has K elements, and evaluate the resulting set of positions $\vec{\Omega}_{LK} = \{\vec{\Omega}_L, \vec{\Omega}_K\}$ for a certain condition. The number of combinations is given by the binomial coefficient $C = \binom{Q}{K} = \frac{Q!}{(Q-K)!K!}$.

Pre-selection of point candidates

The number of different combinations increases rapidly with the number of point candidates and therefore has an enormous impact on the computational complexity of the algorithm. To keep the amount of candidate points adequately low, the surface of the sphere is divided into areas where the given input set $\vec{\Omega}_L$ has large spaces between points and areas of high point density. To do so, we assign every point Ω on the surface with a angular distance value to the closest point of $\vec{\Omega}_L$:

$$d(\Omega) = \min_{\Omega_l = \Omega_1, \cdots, \Omega_L} |\Omega_l - \Omega| \tag{1}$$

The resulting mapping for an example random point distribution $\vec{\Omega}_L$ can be seen in Fig. 2a. Based on this mapping, we can select points from a dense basis grid, e.g. a grid as proposed in [FM99], that lie above and below a certain threshold, e.g. the average distance. The points below this threshold are considered to be no suitable candidates since they are too close to an originally given point. So the candidate point set is reduced to the values above the threshold, as can be seen in Fig. 2b. The threshold could also be dynamically adjusted in regards to the desired number of candidate points.

Test criteria 1: Matrix condition number

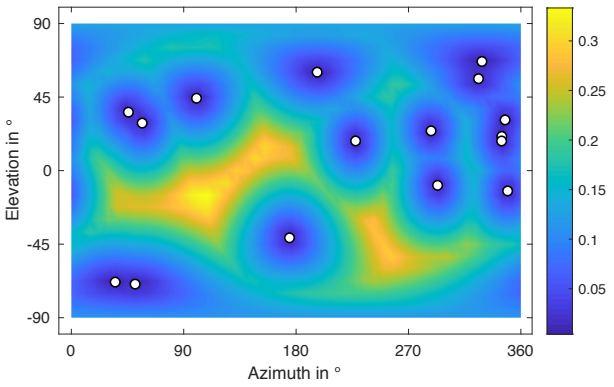
It is common in spatial audio processing to decompose a function like a sound pressure distribution on a sphere with orthogonal basis functions called spherical harmonics (SH). This process is called the spatial Fourier transform (SFT). If the function is sampled at arbitrary points on the sphere the SFT can be performed by a least-squares approximation with the use of a spherical harmonic matrix (SHM) Y, more precisely with its pseudoinverse $Y^\dagger$[Raf15]:

$$f = Y f_{nm} \tag{2}$$

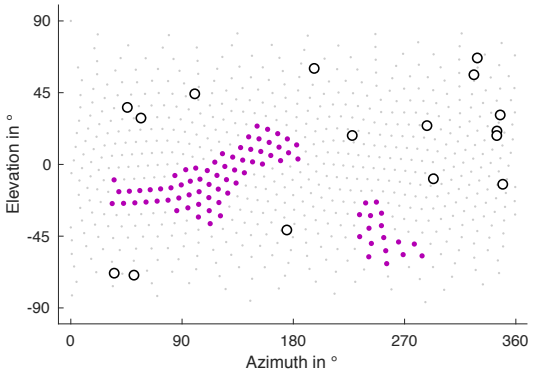
$$f_{nm} = Y^\dagger f \tag{3}$$

where f are the function values sampled at L positions and f_{nm} are the $(N+1)^2$ SH coefficients with N as the SH order.

Y itself is of dimensions $L \times (N+1)^2$, each row contains the spherical harmonics $Y_n^m(\Omega)$ up to order N for one



(a)



(b)

Figure 2: **Pre-selection of points:** (a) the nearest neighbour distance for every position on the grid in relation to a 15 point random grid (b) based on the function, the 80 most distant points are selected from a predefined underlying grid, in this case a 676 point Fliege/Maier grid

sampling position Ω_l.

$$Y = \begin{bmatrix} Y_0^0(\Omega_1) & Y_1^{-1}(\Omega_1) & Y_1^0(\Omega_1) & \cdots & Y_N^N(\Omega_1) \\ Y_0^0(\Omega_2) & Y_1^{-1}(\Omega_2) & Y_1^0(\Omega_2) & \cdots & Y_N^N(\Omega_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_0^0(\Omega_Q) & Y_1^{-1}(\Omega_Q) & Y_1(\Omega_Q) & \cdots & Y_N^N(\Omega_Q) \end{bmatrix} \tag{4}$$

Different sampling schemes provide varying numerical stability for the SFT of a particular order N [Zot09]. A common indicator for the stability is the condition number κ of the SHM [RH17], defined in the euclidean way as the product of the L2-norms of the SHM and its inverse:

$$\kappa(Y^\dagger) = \kappa(Y) = \|Y\|_2 \left\|Y^{-1}\right\|_2 \tag{5}$$

The lowest possible condition number $\kappa = 1$ is only achieved for optimal grid configurations.

Since the SHM for a particular order only depends on the number and the arrangement of sampling positions, and not on the function data itself, κ is well suited as a quality indicator of a sampling grid. When comparing grids with the same number of points, a lower condition

number indicates a more even distribution of sampling points around the sphere.

When transferred to the original problem of adding sampling positions, the criterion to be tested for is the matrix condition number κ. κ is derived from the SHM for each joint point set $\vec{\Omega}_c = \{\vec{\Omega}_L, \vec{\Omega}_{Kc}\}$. The iterative search can be expressed as a minimisation problem for C combinations of Ω_K:

$$\min_{\Omega_{K1},\cdots,\Omega_{KC}} \kappa(Y(\Omega_L, \Omega_{Kc})) \tag{6}$$

Due to the vertical alignment of the sampling positions inside a SHM, we can compute the SHM's for $\vec{\Omega}_L$ and $\vec{\Omega}_K$ separately, so only the SHM for the new points has to be computed at every iteration step. Then, the condition number is calculated for the vertically joint SHMs:

$$\min_{\Omega_{K1},\cdots,\Omega_{KC}} \kappa(\begin{bmatrix} Y(\Omega_Q) \\ Y(\Omega_{Kc}) \end{bmatrix}) \tag{7}$$

Test criteria 2: Minimum point distance

An even point distribution of a certain number of points on a sphere can be asymptotically achieved by solutions to the *hard-spheres problem* [SK97]. The idea is to place points around a sphere in a way that the smallest distance between any of the points is as large as possible.

Regarding the problem of adding sampling positions Ω_K to a given set of sampling positions $\vec{\Omega}_L$, we can evaluate $\vec{\Omega}_{LK}$ in this manner.

For every combination of new points $\vec{\Omega}_{Kc}$, the smallest distance between any of the points (in the combined point set) is determined.

$$m(\vec{\Omega}) = \min_{1\leq i,j\leq M} |\Omega_i - \Omega_j| \tag{8}$$

Since the distance between the existent point does not vary, it is sufficient to evaluate the distances from the new points to the existing points and to the other new points. This minimum distance $m(\vec{\Omega})$ is the criterion to be tested for and should be maximised.

$$\max_{\Omega_{K1},\cdots,\Omega_{KC}} m(\vec{\Omega}_{Kc}) \tag{9}$$

Weighted distribution of sampling points

For for most binaural reproduction environments, the positions around the horizontal plane are of most interest. A higher density is desired around the horizontal plane while the density towards the north and south pole are becoming less important. To compensate this, the algorithm should employ a weighting as a function of the elevation distance from the horizontal plane.

The idea is to apply a scalar weighting factor w to every point candidate depending on its elevation θ:

$$w(\theta) = 1 - \left|\frac{2\theta}{\pi}\right| \qquad \theta = -\frac{\pi}{2}, \cdots, \frac{\pi}{2} \tag{10}$$

So every combination $\vec{\Omega}_K$ of N new points receives a weighting W with the average sum of the N weights of its candidates:

$$W(\vec{\Omega}_K) = \frac{1}{N}\sum_{k}^{N} w(\theta(\Omega_k)) \tag{11}$$

Thus, Eq. 9 becomes

$$\max_{\Omega_{K1},\cdots,\Omega_{KC}} m(\vec{\Omega}_{Kc}) W(\vec{\Omega}_{Kc}) \tag{12}$$

By choosing a suitable weighting function alternatively to Eq. 10, a desired point distribution can be achieved.

Results

Both algorithms were implemented in Matlab and tested for computing time as well as the effect of improvement. The tests covered various combinations of the candidate point set density, the number of candidate points after pre-selection and the number of desired additional points.

The computing time depends on the number of desired points as well as the number of candidate points. Fig. 3 shows a comparison between the two presented algorithms. The results display clearly that the point distance algorithm outperforms the SHM condition algorithm.

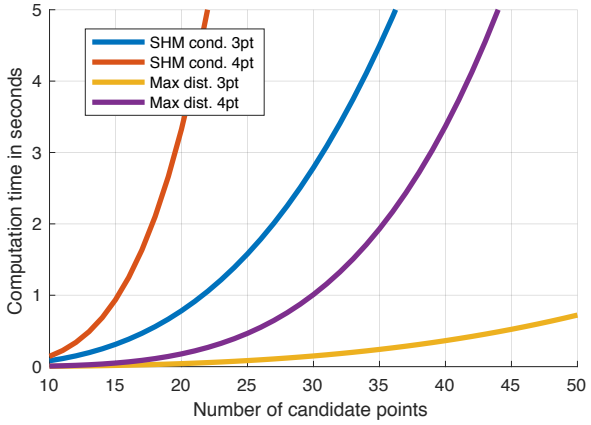


Figure 3: Computing time dependent on the number of point candidates, for a fixed number of desired additional points $k = 3$ and $k = 4$, performed on a 2019 MacBook Pro (2,3 GHz Intel Core i9)

Fig. 4 shows 4 additional points for a random uniformly distributed sample point set, obtained with each of the two algorithms. Even though there are some minor variances in point placement, both algorithms delivered similar results for improving the overall quality of an HRTF dataset.

To quantify the quality improvement for actual HRTF data, we simulated a 15 point HRTF measurement by obtaining HRTFs from a dense reference set [Ber13] for 15 random directions. This 15-point set was extended by the two algorithms with 4 additional points and their corresponding HRTFs. Then we upsampled the random set

and the extended sets with a third-order SFT to obtain sets on the positions of the reference grid. To compare the resampled sets to the reference set, we calculated the difference in spectral magnitude and averaged over all sampling positions of a set. This process was repeated for several times with different random grids, Fig. 5 shows the averaged results for 100 trials. Both approaches delivered similar results and improved the random grid.

Note that the 4 additional sampling points were calculated with 30 candidate positions, obtained from an underlying 324 point (17th order) Fliege/Maier grid, with a computation time of about 1 second for maximum point distance approach and 19 seconds for the SHM condition number approach.

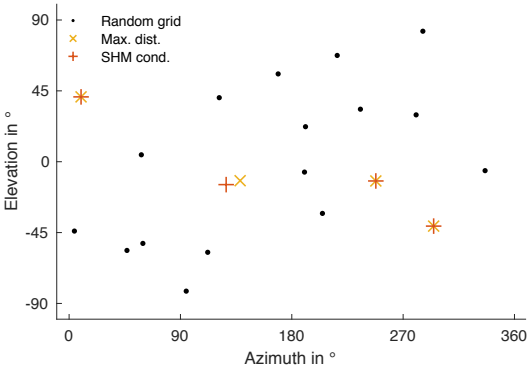


Figure 4: Additional points to an exemplary random sample grid, computed by the two variants of the algorithm.

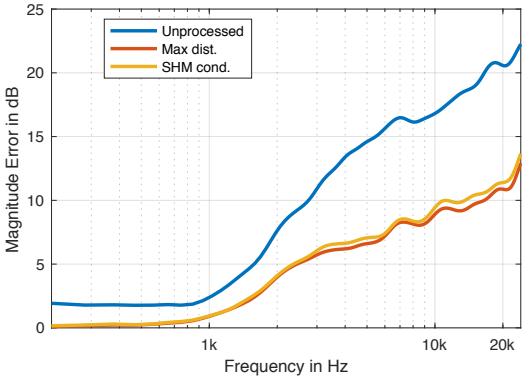


Figure 5: Magnitude error between the upsampled sparse HRTF sets and the dense reference HRTF set, averaged over all sampling positions and over several trials. Only left part of an HRTF is displayed.

Conclusion

In this work, we introduced an approach with two variants for finding additional sampling positions to an arbitrary sampling grid, based solely on the grid positions itself. Furthermore, we showed how to increase computing efficiency by preselecting the possible candidates. The results showed that it is possible to reduce the error caused by very sparse HRTF sets by adding a few points to suitable positions, even with a small number of candidates and therefore a small amount of computing time. This improvement is beneficial during sparse self-driven HRTF measurements with non-optimal position distribution, giving the user a quick improvement without the need for any evaluation of the actual measured data.

The pre-selection of candidate points requires some care in terms of selecting the right angular resolution of the underlying candidate grid. If the resolution is too low, the overall error improvement might be suboptimal. If the resolution is too high, the areas covered by the candidates is too small, giving the potential risk of missing out an optimal position in the resulting points.

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